2023

(May/June)

ECONOMICS

(Honours)

(Mathematics for Economists)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Distinguish between equation and identity with suitable examples. 3

(b) Given the universal set U as $U = \{a, b, c, d, e, f\}$ and $A = \{b, c, e\}$, $B = \{a, c, d\}$. Find—

(i) $(A^c - B^c)^c$

(ii) $(A \cup B^c)^c$

(iii) $(A \cap B^c)^c$

(iv) $(A \cap A^c)^c$ 2×4=8

(c) In a group of 65 consumers, 50 of them consume apple while 20 of them consume both apple and orange. How many of them consume—

(i) orange;

(ii) orange only? 3+1=4

2. *(a)* What are simultaneous linear equations? How can these equations be used in solving economic problems? Give one example. 4

(b) Determine the degree of homogeneity of the following functions : 3+3=6

(i) $f(x, y) = x^3 - 5xy^2 + y^3$

(ii) $f(L, K) = [3L^2 + 5K^2]^{1/2}$

(c) The prices and quantities demanded for a particular commodity during two different periods are as follows :

	Prices	*Quantities*
Period–1	₹ 5	12 kg
Period–2	₹ 8	6 kg

Obtain the linear demand function. What would be the quantity demanded if the price was ₹ 9? 4+1=5

UNIT—II

3. *(a)* Distinguish between diagonal matrix and identity matrix with suitable examples. Show that identity matrix is always idempotent. 3+2=5

(b) If

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -6 & 5 \\ 3 & -4 \end{bmatrix}$$

then find the matrix D such that $5A - 4B - 7D = 0$. 5

(c) What is determinant of a matrix? With suitable example, show that if two adjacent rows (or columns) of a given determinant are interchanged, then the given determinant gets multiplied by –1. 2+3=5

4. *(a)* If

$$A = \begin{bmatrix} 3 & -2 & -1 \\ 2 & 1 & 0 \\ -3 & 0 & 5 \end{bmatrix}$$

then prove that $A^{-1} \cdot A = I$. 6

(b) Solve the following equations by Cramer's rule : 5

$$\frac{x}{3} - \frac{y}{6} = 1$$

$$\frac{x}{4} + \frac{y}{3} = 1$$

(c) What is Leontief input-output model? State Hawkins-Simon conditions associated with this model. 1+3=4

UNIT—III

5. (a) Under what conditions, a function $f(x)$ is said to be continuous at the point $x = a$? Show that the following function is continuous at $x = 3$: 1+4=5

$$f(x) = \begin{cases} x^2 - 5; & 0 < x < 3 \\ x + 1; & 3 < x < 6 \\ 2x - 2; & \text{otherwise} \end{cases}$$

(b) Evaluate the following limits (any *two*) : 2×2=4

(i) $\underset{x \to a}{\text{Lt}} \dfrac{x^6 - a^6}{x^4 - a^4}$

(ii) $\underset{n \to \infty}{\text{Lt}} \dfrac{3n^2 - 5n^{-1}}{4n^2 - 6n^{-2}}$

(iii) $\underset{h \to 0}{\text{Lt}} \dfrac{(x+h)^3 - x^3}{5h}$

(iv) $\underset{x \to 2}{\text{Lt}} \dfrac{x^3 - 8}{x^2 - 6x + 8}$

(c) Find $\dfrac{dy}{dx}$ of the following functions (any *two*) : 3×2=6

(i) $y = 3x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} + 10$

(ii) $y = \log(x^2 - \sqrt{6 - x} + 1)$

(iii) $x + y + (x + y + 5)^3 = 0$

(iv) $y = (x)^{\frac{1}{x}}$

6. (a) If

$$y = ax^2 + \frac{a}{x^2}$$

then prove that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 4y$$ 5

(b) If $u = x^3 - 4x^2y + y^3$, then show that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$ 3

(c) (i) If $z = \sqrt{x + y}$, then find dz.

(ii) If

$$z = \log\left(\frac{x - y}{x + y}\right)$$

then find dz.

where dz is the total differential of z. 3+4=7

UNIT—IV

7. *(a)* Determine the maximum/minimum of the following function : 6

$$y = x^3 - 12x + 30$$

(b) The demand function is given by $x = \frac{30}{P+6}$. Determine the price elasticity of demand if the price was ₹ 4. Also interpret the result. 6

(c) If MR = ₹ 26 and price elasticity of demand is 3, then find AR. 3

8. *(a)* The total cost function of the firm is $C = 4x - x^2 + 2x^3$. Show that when AC is minimum, AC = MC. 6

(b) The demand function and total cost function are the following :

$$q = 100 - P$$

$$C = \frac{1}{3}q^3 - 7q^2 + 111q + 50$$

Determine the profit maximizing level of output (q). Also write down the value of profit and the corresponding price (P) at this level of output. 6+2+1=9

UNIT—V

9. *(a)* What is integration? Why is there always a constant of integration? 1+2=3

(b) Evaluate the following integral (any *two*) : 3×2=6

(i) $\int x e^{-x} dx$

(ii) $\int \frac{\log x}{x} dx$

(iii) $\int (6x-5)\sqrt{3x^2 - 5x + 1}\, dx$

(iv) $\int x^2 e^{3x} dx$

(c) The demand and supply functions are $P_d = 26 - 5q$, $P_s = 4q + 8$. Find consumer's surplus. 6

10. *(a)* Explain briefly the concepts of definite and indefinite integral with examples. 2+2=4

(b) Evaluate the following integral : 5

$$\int_{-a}^{a} (a^2 - ax + x^2)\, dx$$

(c) The supply function is given by $q = \sqrt{p - 16}$. Find the producer's surplus if the price was ₹ 20. 6

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